*Class Notes:Doppler Shift Studied With a Rotating Sound Source.*

*A rough draft description of the measurements and their interpretation for Assignment 8.*

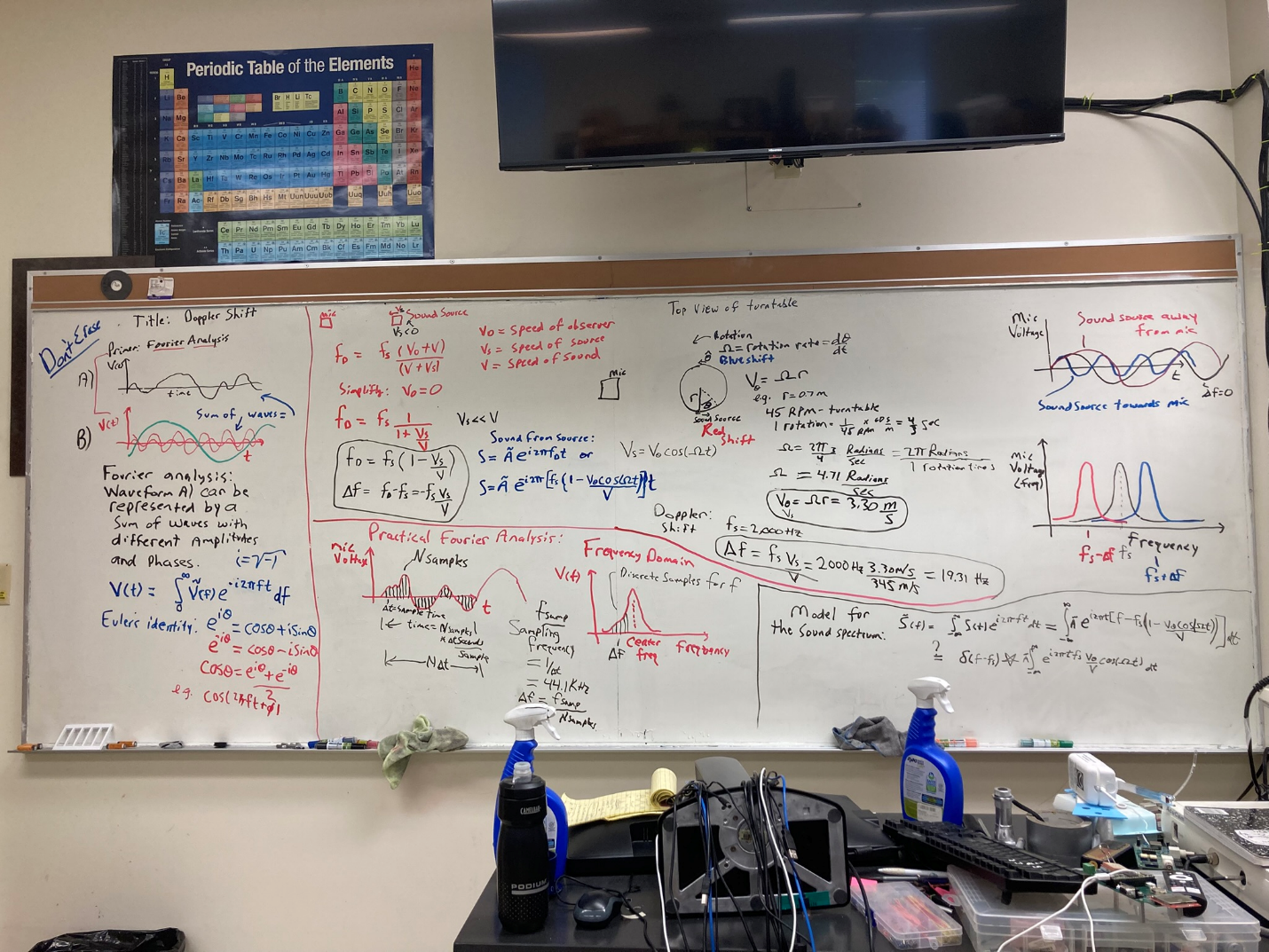


Figure . Class notes for rotating sound source measurements.

*Our observed spectrum made using Audacity.*

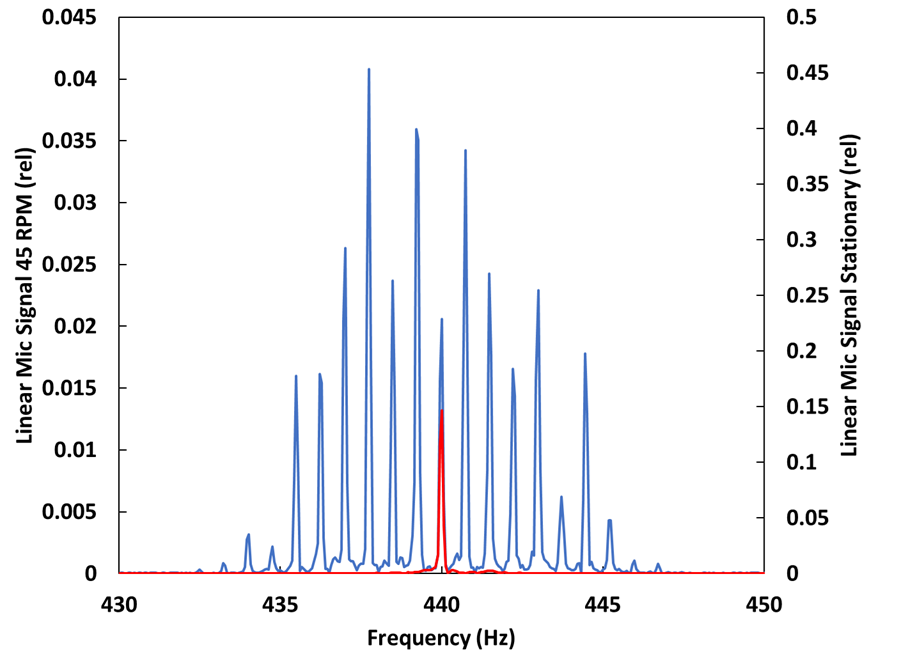


Figure . Observed spectra for the stationary sound souce (red) and the rotating sound source (blue).

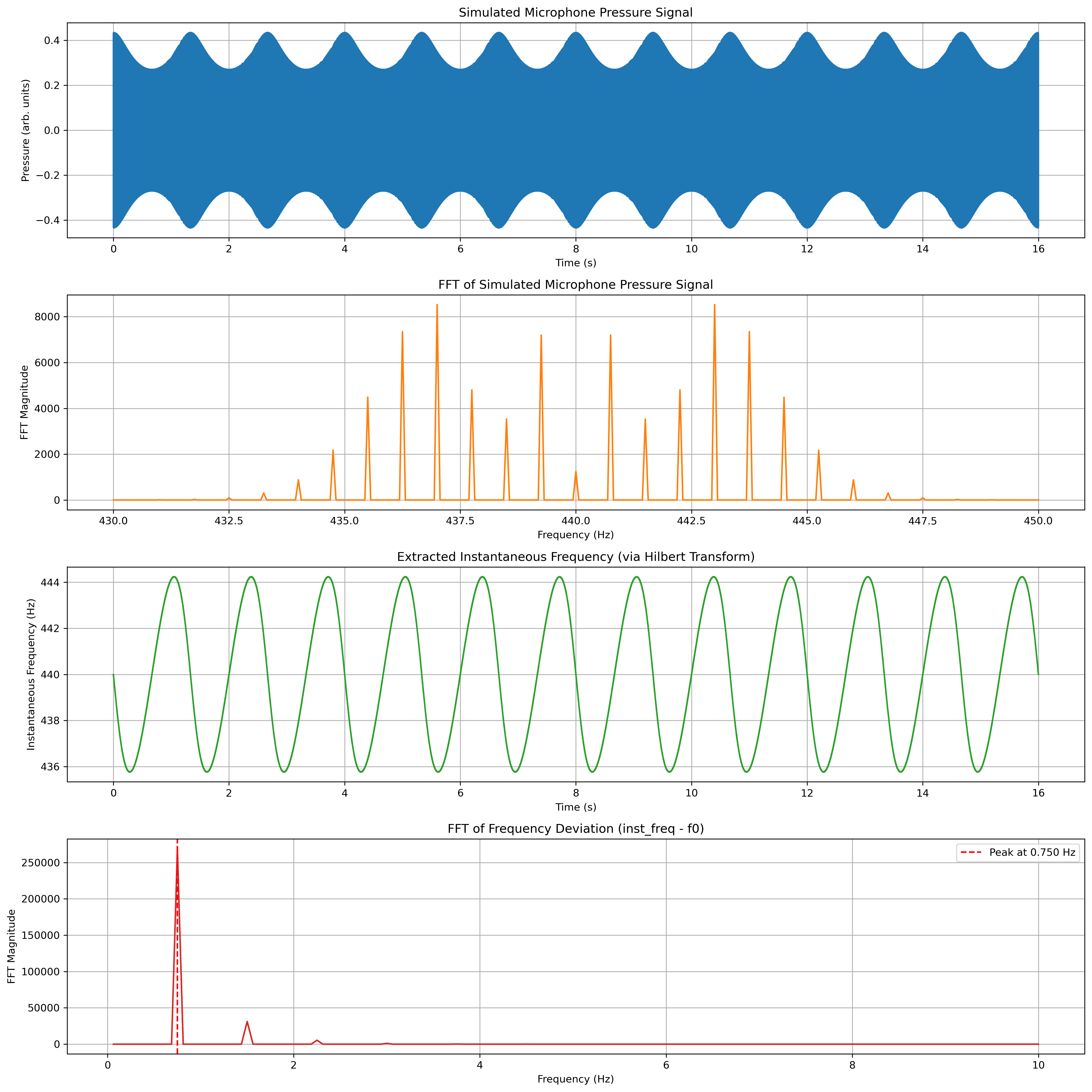


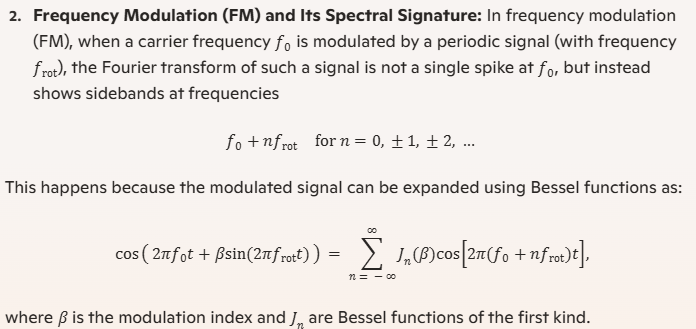
Figure . Modeled rotating dish measurements for the 440 Hz source. The model is available [here](https://www.patarnott.com/atms360/general/class2025/Assignment7_DopplerEffect/HilbertTransform_RotatingSoundSourceModelAndRetrieval_WithPsignalFFT.py). <https://www.patarnott.com/atms360/general/class2025/Assignment7_DopplerEffect/HilbertTransform_RotatingSoundSourceModelAndRetrieval_WithPsignalFFT.py>.

*Some questions to AI:CoPilot:*

*Why do rotating sound sources have frequency spectra with maxima separated by the rotational frequency?*

When a sound source rotates, its motion produces a periodic modulation in the signal received by a fixed observer. This modulation—most notably due to the Doppler effect—causes the instantaneous frequency (and sometimes the amplitude) of the signal to vary in a periodic fashion. The key points are as follows:

1. **Periodic Doppler Modulation:** As the sound source rotates, the component of its velocity toward or away from the observer changes sinusoidally (or nearly so). For a source emitting a pure tone at frequency , the observed instantaneous frequency becomes  
     
   where with frequency is the rotational frequency and is the modulation amplitude determined by the maximum radial Doppler shift.



1. **Interpretation of Spectral Maxima:** The peaks or maxima in the frequency spectrum appear at intervals of because the modulation is periodic with that frequency. Each “sideband” in the spectrum corresponds to a harmonic of the rotation rate, and the spacing directly reflects the fundamental period of the modulation—that is, the rotation period.

In summary, the rotation of the sound source causes its signal to be frequency modulated (due to the periodic changes in relative motion). The frequency spectrum of an FM signal naturally contains components (sidebands) that are spaced by the modulation frequency—and in this case, that modulation frequency is exactly the rotation rate of the source.

This is why rotating sound sources exhibit spectra with maxima (or sidebands) separated by the rotational frequency.

Would you like further details on the mathematics behind FM sidebands or examples of how this effect is used in practical signal processing?

yes, further details

Below is a more in‑depth explanation with mathematical details and derivation of why a rotating sound source produces a frequency spectrum whose peaks are separated by the rotational frequency.

**1. The Basic Model**

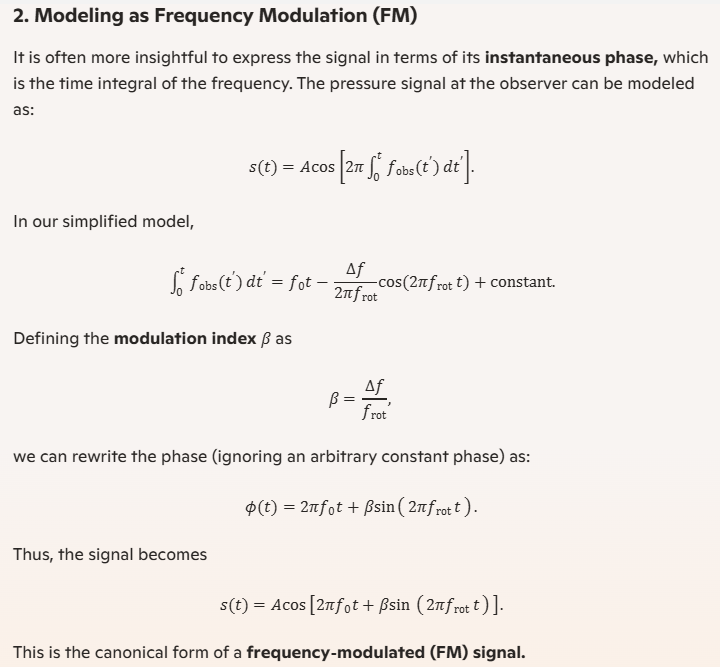
Consider a sound source emitting a pure tone of frequency

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When the source rotates, its motion relative to a fixed observer varies periodically. Because of this periodic motion, the Doppler effect produces a time‑varying frequency shift. For simplicity, assume that the instantaneous observed frequency is given by

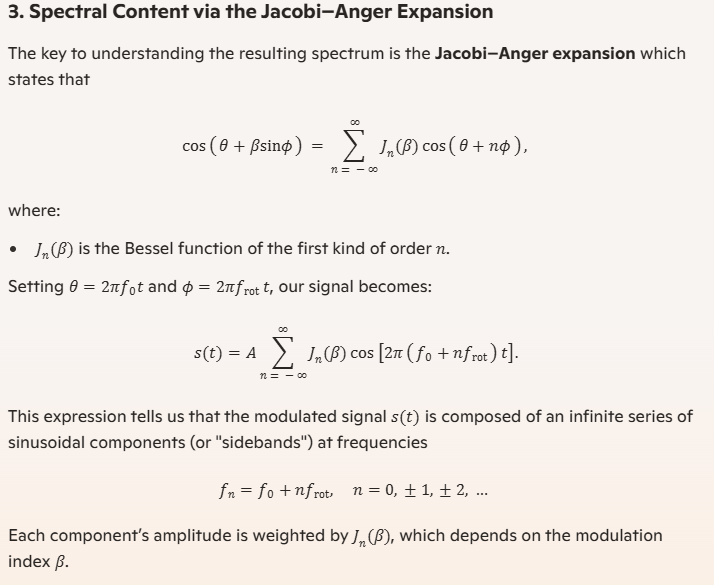
where:

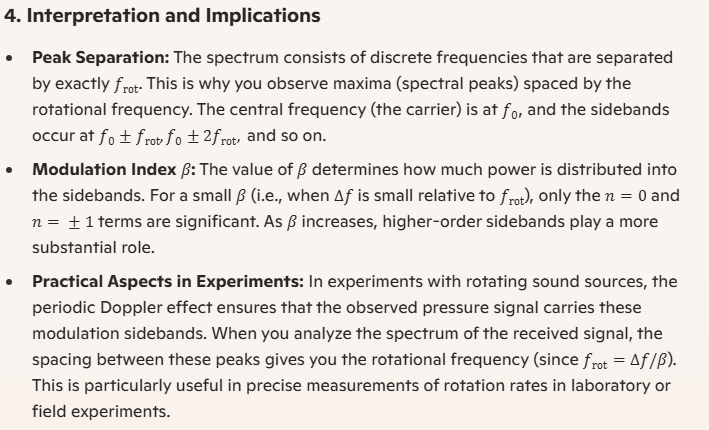
* is the peak frequency shift (determined by maximum radial velocity and the Doppler relation), and
* is the rotational (or modulation) frequency—i.e., the number of rotations per second.

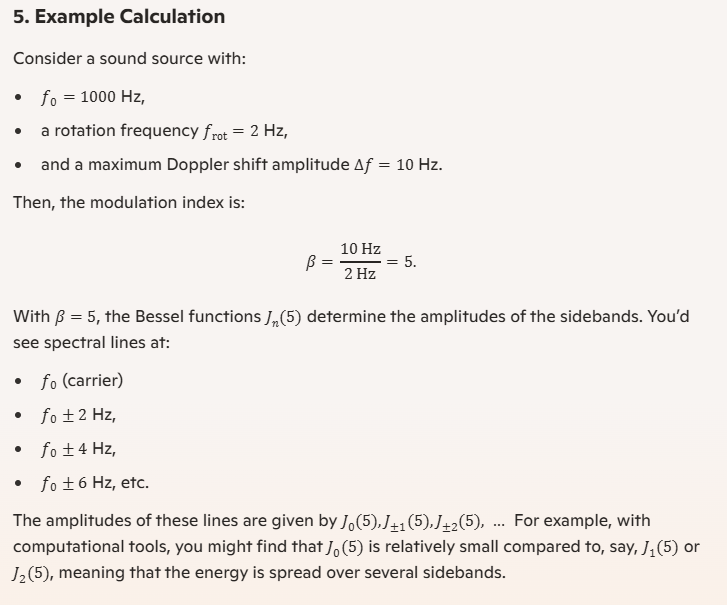
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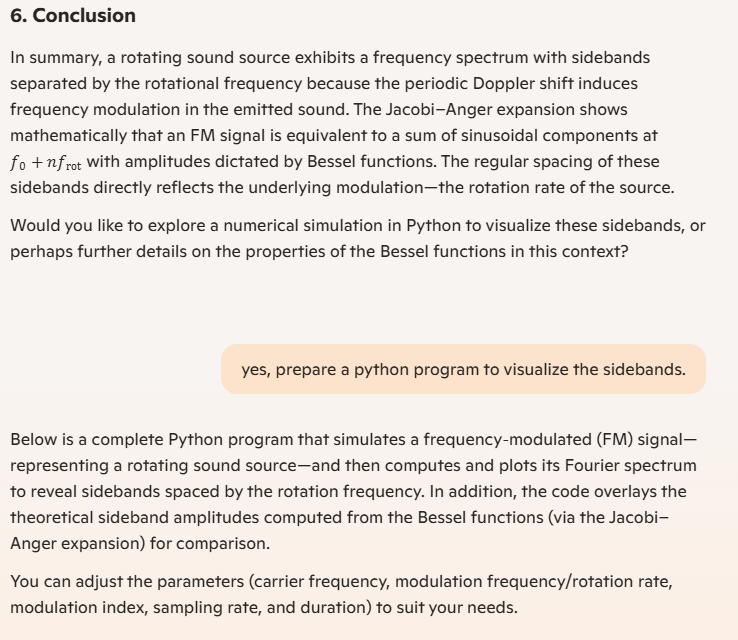
For our set up,

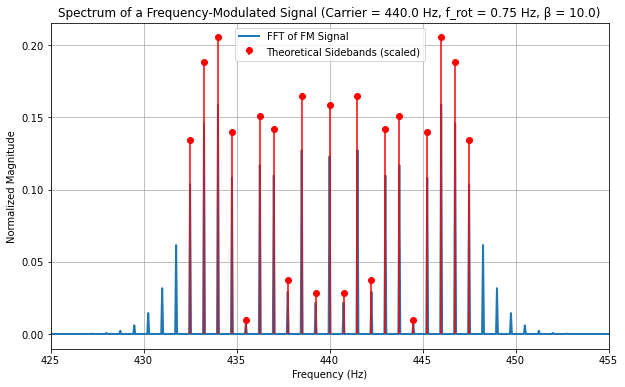
Changing the frequency of the sound at the speaker should increase the modulation index and provide more sidebands in the frequency spectrum.

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Below is a complete Python program that simulates a frequency‐modulated (FM) signal—representing a rotating sound source—and then computes and plots its Fourier spectrum to reveal sidebands spaced by the rotation frequency. In addition, the code overlays the theoretical sideband amplitudes computed from the Bessel functions (via the Jacobi–Anger expansion) for comparison. The example above shows it.

python

import numpy as np

import matplotlib.pyplot as plt

from scipy.special import jv # Bessel functions of the first kind

# ==============================

# Parameters for the FM Signal

# ==============================

f0 = 440.0 # Carrier frequency in Hz

f\_rot = 0.75 # Rotation (modulation) frequency in Hz (sidebands will be spaced by 2 Hz)

beta = 5.0 # Modulation index (beta = Δf / f\_rot); higher beta spreads power to more sidebands

fs = 8000 # Sampling frequency in Hz (must be high enough to capture the carrier)

T = 20.0 # Total time duration in seconds; longer time gives better frequency resolution

# Time vector

t = np.arange(0, T, 1/fs)

# ==============================

# Generate the FM Signal

# ==============================

# The signal is given by:

# s(t) = A cos[2π f0 t + β sin(2π f\_rot t)]

# This is the canonical form of a frequency-modulated waveform.

A = 1.0 # amplitude (arbitrary units)

s = A \* np.cos(2 \* np.pi \* f0 \* t + beta \* np.sin(2 \* np.pi \* f\_rot \* t))

# ==============================

# Compute the FFT of the Signal

# ==============================

N = len(s)

S = np.fft.fft(s)

f = np.fft.fftfreq(N, d=1/fs)

# Shift FFT so that zero-frequency is in the center.

S\_shifted = np.fft.fftshift(S)

f\_shifted = np.fft.fftshift(f)

# Normalize the magnitude by the number of samples

mag = np.abs(S\_shifted) / N

# We are interested in the spectrum near the carrier frequency.

# Define a mask to zoom in around f0 (say f0 ± 50 Hz)

mask = (f\_shifted >= f0 - 50) & (f\_shifted <= f0 + 50)

# ==============================

# Theoretical Sideband Amplitudes

# ==============================

# From the Jacobi-Anger expansion, an FM signal can be written as:

# s(t) = A sum\_{n=-∞}^{∞} J\_n(β) cos[2π (f0 + n f\_rot) t]

# where J\_n(β) is the Bessel function of order n.

# We will consider orders from -10 to 10.

orders = np.arange(-10, 11)

theoretical\_freqs = f0 + orders \* f\_rot

# Compute the theoretical amplitude for each sideband.

# Note: In an infinite-length signal, the amplitude at each sideband is A \* J\_n(beta).

theo\_amplitudes = A \* np.abs(jv(orders, beta))

# Because the FFT amplitude (from a cosine signal) may be split between positive and negative

# frequency components, we need to scale the theoretical amplitudes for proper comparison.

# Here, we simply scale so that the theoretical central peak (n=0) matches the FFT's central amplitude.

central\_idx = np.where(np.isclose(orders, 0))[0][0]

scale\_factor = np.max(mag[mask]) / theo\_amplitudes[central\_idx]

theo\_amplitudes\_scaled = theo\_amplitudes \* scale\_factor

# ==============================

# Plotting the Spectrum and Theoretical Sidebands

# ==============================

plt.figure(figsize=(10, 6))

plt.plot(f\_shifted[mask], mag[mask], label='FFT of FM Signal', lw=2)

plt.xlabel('Frequency (Hz)')

plt.ylabel('Normalized Magnitude')

plt.title('Spectrum of a Frequency-Modulated Signal (Carrier = {:.1f} Hz, f\_rot = {:.1f} Hz, β = {:.2f})'.format(f0, f\_rot, beta))

plt.grid(True)

# Overlay the theoretical sideband positions and amplitudes

plt.stem(theoretical\_freqs, theo\_amplitudes\_scaled, linefmt='r-', markerfmt='ro', basefmt=" ", label='Theoretical Sidebands (scaled)', use\_line\_collection=True)

plt.legend()

plt.xlim(f0 - 15, f0 + 15)

plt.show()